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"NOTES FROM THE STOCKPILE SEMINAR"

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A seminar was held at NPS in the Spring of 1984, the object being to review some of the models used by the armed services for planning weapon procurement. Most of the effort was spent on the Navy's NNOR and the Air Force's Sabre Mix Methodologies. This report is a summary of the conclusions.

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1. Introduction

Even in an emergency situation, it is difficult to speed up the production rate of sophisticated, modern weapons. The time constant for increasing production rate for many weapons seems to be on the order of a year, whereas major wars are sometimes imagined to last for only several months. For many weapons, in fact, it is not a bad approximation to assume that a major war would have to be fought entirely with war reserve stocks. It might be possible to accomplish a certain amount of geographic redistribution during a war, but not much production.

Given these supposed facts, the following question would seem to be crucial for the yearly POM process: "How should a fixed budget be spent augmenting the current stockpile of weapons so as to maximize the effectiveness of the resulting stockpile?" Operations Research techniques could play an important role in answering the question, since several favorable preconditions exist:

* The question must be asked repetitively.
* Combat modelling must inevitably be involved in assessing effectiveness.
* Lots of data are available that must be taken into account.
* The problem of determining the best stockpile can be interpreted as one of mathematical optimization.

In the Spring of 1984, a "Stockpile seminar" was held at NPS, the object of which was to review whether the crucial question mentioned above actually gets asked, and also to determine the role of Operations Research in answering it. The present document is a record of the deliberations and observations made during and after that seminar.

The seminar was run and attended mainly by faculty in the Operations Research Department, although one of us (Boger) is in the Administrative Sciences Department, and some OR PhD students also participated. The goal of the seminar was to teach each other about and criticize the methods and models used by the several armed services, both for tutorial reasons (this report will be used as a handout) and because some of us are interested in doing further research in what appears to us to be a vital area for the Department of Defense. The seminar was held at the one-hour-per-week level throughout most of the Spring quarter of 1984, climaxed by one all-day session attended by several outside experts. A list of attendees at that session is included as Appendix 1.

We are not the first to conduct a comparative analysis of methods for determining stockpiles. References [1,8,10] are examples of previous studies. The present report is flawed, moreover, in not being comprehensive; the topic is a large one, and there are many areas that we simply didn't look into. In particular, we were unable to find out very much about US Army methods. Nonetheless, we will make some observations that are not contained in previous studies.

Except for the Navy's use of the OPM model (see sec. 2.5), the crucial question above doesn't get asked. Instead, the question is "What stockpile is needed to satisfy our requirements," or sometimes "What is the cheapest stockpile that will satisfy our requirements?" The typical answer to this question is that, even when costs are minimized, the required stockpile is considerably more expensive than is fiscally feasible. Figure 1, for example, shows for a typical weapon the comparison between inventory and the Navy's

"programming objective profile" as determined by the NNOR (sec 2.2). There is clearly a large difference between the two, particularly if the gap is compared to the yearly stockpile increment. One way of resolving the discrepancy between budgets and requirements would be to reassess requirements (possibly also budgets) until feasibility is finally achieved. That is not typically done, however. We are not sure exactly what is done, but the models used to determine the requirements-driven stockpiles in the first place apparently play only a minor role.

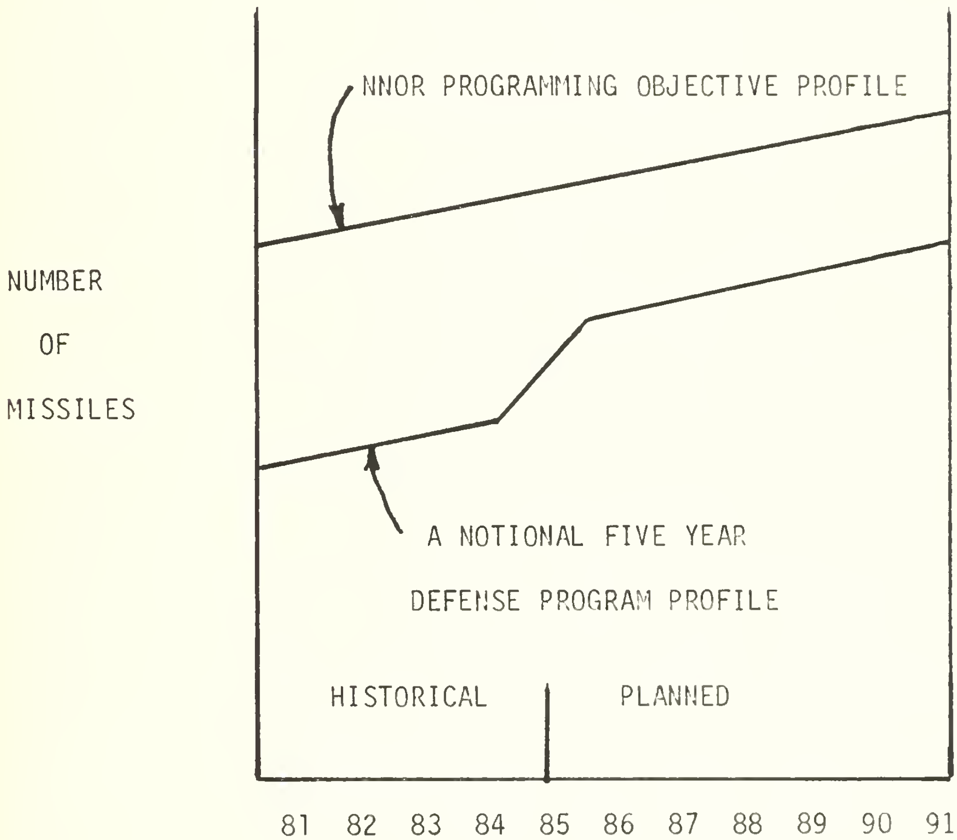

FIGURE 1

There are several reasons, some of them good ones, why the armed services persist in asking what appears to be the wrong question. Our feeling is that the system (the national defense of the USA) would be better off if budget constrained questions were asked in the first place, but we will spare the reader our standard academic rant on the subject. Suffice it to say that if weapons cannot compete against each other for representation in the stockpile, then it is going to be difficult to make the cost/effectiveness tradeoffs that are essential in developing a proper weapons mix. These comments are more related to the budgeting system than to any of the quantitative models used therein, but there are additional weapons mix biases built into the models themselves, as will be seen.

2. Current Navy Methods

The methodologies that are presently being used by the Navy to generate ordnance requirements for the POM are known collectively as the Non-Nuclear Ordnance Requirements (NNOR). The NNOR is composed of two separate approaches to the modelling of combat requirements for major ordnance expendables. The first methodology addresses only the air-to-surface requirements. It is a "level of effort" method, by which is meant that the need for weapons is limited by the number of shooters, rather than the number of targets. The second is a "threat-oriented" methodology wherein the need for weapons is determined mainly by the number of targets. The NNOR generates requirements for all other major ordnance expendables.

2.1 Level-of-Effort Methods (NAVMOR)

Air-to-surface requirements within the NNOR are calculated via three computer models; PHASER, NAVMOR, and MRG. PHASER is the model which generates the numbers of sorties flown and aircraft attrited in various scenarios across several theaters. These outputs from PHASER become inputs for NAVMOR and MRG.

PHASER accepts as inputs such items as carrier employment schedules, aircraft inventories and locations, sortie rates, attrition rates, weather factors, and maintenance factors. A separate PHASER run is required for each aircraft type across all scenarios, theaters, and time. PHASER outputs are generated within fixed time intervals (phases) during which sortie input parameters remain fixed. Sortie rates and attrition rates are adjusted for weather and maintenance effects. PHASER keeps track of the expected aircraft assets during each phase of a scenario, but not the distribution of assets. The output of sorties flown may be decomposed by mission types. Finally, these output estimates are divided into general-purpose weapon sorties and special-purpose weapon sorties. This percentage allocation is made on the basis of subjective judgments of experienced military personnel. The general-purpose weapon sorties are used as inputs to NAVMOR, and the special-purpose sorties are inputs to MRG.

NAVMOR (Navy and Marine Ordnance Requirements) is the computer model used to determine combat requirements for ordnance expendables using PHASER-generated attack sorties for general-purpose weapons. The general methodology used in NAVMOR is that total sorties input to the model are allocated over different prespecified sortie types. A cost competition determines the optimal NAVMOR weapon type for each sortie type defined to be armed with a NAVMOR weapon.

This cost competition is really the heart of NAVMOR, so it is worthwhile looking at it in some detail. The competition takes the form of maximizing the expected number of kills per dollar by selecting optimally the decision variables (w,i,t,n), where i is the number of weapons of type w loaded onto a

sortie that will make n passes using tactic t. The expected number of target kills, $k(w,i,t,n)$, is determined from the Joint Munitions Effectiveness Manual (JMEM), and the expected number of kills per dollar is given by

$$\frac{k(w,i,t,n)}{(i*U(w))+O+(R*P_k(w,t,n))} \tag{1}$$

where

- $U(w)$ = cost per weapon type w,
- O = operations and maintenance cost per sortie,
- R = aircraft and crew cost, and
- $P_k(w,t,n)$ = one pass attrition probability.

Additionally, $P_k(w,t,n)$ is given by

$$P_k(w,t,n) = 1 - (1 - \lambda S(w,t))^n, \tag{2}$$

which the reader will recognize as the formula which applies when a fixed number, n, of passes is made with an attrition probability per pass of $\lambda S(w,t)$. The function $S(w,t)$ comes from relative, judgmental inputs of the form "attrition using tactic t is (say) double that of standard tactics." The attrition scaling factor, λ, is designed to convert such relative judgments into one-pass attrition probabilities. Formulas (1) and (2) are essentially formula (2-23) of reference [6], except that additional parameters determining target type, aircraft type, mission type, weather, and time of day have been suppressed.

After determining (w,i,t,n) in each case based on the cost/effectiveness of formula (1), total attrition can be obtained by summing over the additional parameters mentioned above. This total attrition is forced to meet a predetermined constraint by adjusting the attrition scaling factor λ until the calculated attrition is judged to be realistic. This procedure permits expert attrition judgments to be made on a relative scale while preserving an

absolute attrition constraint. This desirable feature is achieved at some cost, however, as will be discussed further in section 2.3.

The Miscellaneous Requirements Generator (MRG) is the computer model used to calculate all level-of-effort combat ordnance requirements not generated by NAVMOR. MRG calculates requirements on an ordnance item basis with no competition between items for the arming of sorties. For each item, the special-purpose sortie output from PHASER is used to find the sortie estimates for any aircraft type compatible with the item. These sorties are then allocated over mission and operation type based upon subjective military judgment. The basic computational concept of MRG is that combat requirements are equal to the product of total sorties flown, the fraction of those sorties on which the item is used, and the number of items expended per utilization sortie. Accumulation of these requirements over all sortie types yields total combat requirements for that item.

2.2 Threat-Oriented Methods (the THREAT model)

The Navy uses a static methodology to calculate combat requirements for threat-oriented environments. These environments include such missions as AAW, ASUW, and ASW. By changing certain elements of the underlying methods, the same methodology can apply to all three types of missions.

The threat-oriented methodology begins by partitioning the threat; that is, it separately treats each ordnance item for which combat requirements are to be generated. The methodology uses a generalized "shoot-look-shoot" policy in which independent salvos, not single shots, are fired sequentially at sequentially arriving targets.

This policy results in a number of required salvos that is a geometric random variable with a mean equal to the reciprocal of the salvo kill

probability. It was presumably this feature that the Secretary of the Navy had in mind when he said:

> "It took us the first full year of the Administration to turn around the totally unrealistic peacetime planning models that the analytical community had foisted on the operators. You could only buy two torpedoes for every target in the Soviet fleet that was worth a torpedo, because you had say a 55% or 65% kill probability, and so two gave you over 100% and, therefore, you could not buy any more. That's the situation we were in; it was totally unrealistic."
> -John Lehman, Armed Forces Journal, 1983

Note that it is the low performance weapons that are emphasized in the threat-oriented method, since many weapons are required to do the job. This tendency to use many weapons is to some extent mitigated by truncating the geometric distribution to account for the inability of platforms to fire at particular targets, such as fast-movers, until they obtain a kill. All probability remaining at salvo values larger than the maximum depth-of-fire value is lumped with that at the maximum depth-of-fire.

One problem that the Navy must face in stockpiling weapons is the maldistribution of targets over platforms. The obvious analytic assumption that each target is independently equally likely to encounter each platform is unduly optimistic, since historical experience is that targets are much more concentrated than that. The assumption actually made is that target encounters follow a "contagion" model wherein the probability that each target encounters a given platform is proportional to the number of targets already encountered by that platform. The parameter β is each platform's "initial experience," with $\beta = 0$ corresponding to a situation where some unlucky platform encounters every target, and $\beta = \infty$ corresponding to the earlier rejected assumption of perpetual equal likelihood. Although there is no evidence that this contagion model provides an especially good fit to history, it is no less plausible than other models, and there is enough

historical evidence to establish that the best value for β is approximately 1.0. When there are many targets and platforms, $\beta = 1$ corresponds to a geometric distribution for the number of targets encountered by a platform, whereas the case $\beta = \infty$ would correspond to a Poisson distribution. The nature of this distribution is important because the Navy's policy is to acquire enough weapons (budgets permitting) to make the probability that the typical platform does not run out exceed some confidence level of approximately .9. The required inventory is quite sensitive to this confidence level when the distribution is geometric.

The calculation of ordnance expenditures combines the distributions of the number of salvos expended to kill a particular number of targets and the number of targets encountered by a platform. The result is the distribution of the maximum number of salvos expended by any single platform. This distribution is then modified to reflect the existence of false targets, and finally an initial allowance for each combatant is calculated to satisfy the aforementioned confidence level.

When the estimate of a platform's initial allowance is larger than its magazine capacity, replenishment is necessary. The replenishment model used here is a reservoir model. The reservoir draws down under heavy demand and refills to its limit during light demand. A deterministic refill size, the average refill, is used for replenishment. Replenishment demand for a group of platforms is obtained by aggregating individual demands while noting that the individual demands are correlated because of the fixed number of targets in the pool. Groups of platforms place demands on station ships which then place demands on depots. Failure to supply these demands increases the vulnerability of platforms and serves as a measure of effectiveness for the risk of a reduced inventory. The measure of effectiveness of the THREAT model

may be precisely stated as, at any selected level of confidence, (1) platforms receive enough ordnance, up to magazine capacities, to assure they do not run out; (2) given that platform requirements exceed their capacities, the logistics ships are loaded out, up to capacities, to assure that platforms do not run out; and (3) given that platform requirements exceed logistic ship capacities, the ashore depots have sufficient stocks to assure that platforms do not run out. The model assumes no delays and no ordnance losses in resupply.

The foregoing discussion of requirements determination in the threat-oriented model makes no mention of attrition to friendly platforms. There is nonetheless a sense in which friendly attrition is built into the model; this may even be intuitively reasonable, since one effect of attrition ought to be to concentrate targets on certain platforms (the ones still surviving) as predicted by the model. Here is the sense in which attrition is "built-in": Imagine first that a certain number (a) of "attritions" are mixed in with the (k) targets. A platform unlucky enough to be chosen by an attrition is immediately sunk; unlike targets, attritions cannot be shot down. The sequence in which attritions and target kills appear is governed by a Markov chain with state (k,a). Conceptually, the model starts at event k+a and works back to event 1 in the representation

$$(k,a) \xrightarrow{K} (k-1,a) \xrightarrow{A} (k-1,a-1) \rightarrow \ldots \rightarrow (1,0) \xrightarrow{K} (0,0),$$

where K denotes the occurrence of a kill and A denotes the occurrence of an attrition. The model assumes the following conditional probabilities at any stage:

$$\Pr((k-1,a) \mid (k,a)) = k \,/\, (k+a\beta) \text{ and}$$

$$\Pr((k,a-1) \mid (k,a)) = a\beta \,/\, (k+a\beta).$$

This indicates that, for small β, target kills tend to occur late in the sequence and attritions early. For $\beta = 1$, any sequence of K's and A's is equally likely. A remarkable result (Reference [5], Appendix O) is that as long as (a) is less than the number of platforms, the distribution of the number of kills per platform is independent of (a). In this sense then, attrition is built into the model.

None of these attrition calculations is actually carried out in the threat-oriented model. The fact that the distribution of kills per platform is independent of (a) means that (a) might as well be set to 0; i.e., attrition can be ignored for purposes of calculating weapon allowances as long as the attrition mechanism described above is felt to be probabilistically realistic. There is no evidence that it is realistic. It is not implausible, however, and there is also no evidence to the contrary.

2.3 Observations on NAVMOR

The optimization part of the NNOR is incorporated within NAVMOR, more specifically in the part of NAVMOR where weapons compete on a cost/ effectiveness basis to be loaded on sorties. This is the only part of the NNOR where mathematical optimization can potentially affect the weapons mix, so it is worthwhile looking at it in some detail. We will do so in this section, staying alert to the possibility that the method in use may introduce biases into the weapon selection process.

As this is written, NAVMOR is being run in support of POM 88, with the crucial input from PHASER being the number of sorties of various types that would be generated by all the aircraft the Navy expects to own in 1988 in the

process of fighting a hypothetical war that begins then. PHASER would also provide similar inputs for wars beginning in later years, but the "outyears" are taken less seriously than 1988 in POM 88. These sorties are simply provided as gross totals; time is not represented explicitly in NAVMOR. There is no specific input list of targets, which is in keeping with the level-of-effort assumption that this is the part of warfare where weapon consumption is limited by the number of shooters, rather that the number of targets. The operating assumption is that the environment is "target rich", so that there is no possibility of running out of targets regardless of which weapons are carried. The object is not to kill all the targets (as it is in the threat-oriented case), but rather to kill targets cheaply. The general idea is to apply the input sorties as efficiently as possible, sum up the weapons required, and use the resulting total (augmented by peacetime consumption) as guidance in drafting the POM.

"As efficiently as possible" means that weapons and tactics are chosen to maximize bang per buck as given by (1). The numerator k of (1) is described in [6] as "the JMEM determined expected kills (in n passes using tactic t to deliver i weapons of type w)". There are actually several ways to derive that quantity from information in the JMEM, since the JMEM tabulates only p(i) = "the probability of killing a target in one pass with i weapons" (let us agree to suppress dependence on w and t for the moment). To be definite, suppose that we are considering a sortie loaded with 12 weapons to be delivered in 3 passes with 4 weapons in each pass. Let $Q = 1-\lambda S(w,t)$ be the over-the-target attrition probability; this is the same expression that is involved in the denominator of (1). There are then at least four expressions that fit the

above English description of the numerator k:

1) $k = 3p(4)$

2) $k = (1-p(4))^3$

3) $k = p(4)(Q+Q^2+Q^3)$

4) $k = p(4)(1+Q+Q^2)$

The first expression corresponds to making three passes at three targets. The second corresponds to making three passes at one target. The third corresponds to making passes at three targets until attrition occurs, with the chance for attrition occurring before each pass. The fourth is the same as the third except that attrition occurs <u>after</u> each pass. There are other possible versions of 2) that include attrition before <u>and</u> after each pass.

In a world where one expression is justified, use of a different one might lead to an unfortunate choice of weapons or tactics. The "powering-up" philosophy of 2), for example, is more likely than 1) (generalized to permit an arbitrary number of passes) to result in a single pass being optimal, and also more likely to result in selection of expensive weapons with high kill probabilities. Expression 4) is more likely than 3) to result in selection of standoff weapons for which Q is large. Expression 1) is more likely than 4) to result in the selection of aggressive tactics and weapons for which Q is small; there is a limit to this argument because Q also appears in the denominator as part of the sortie cost, but there is nonetheless a tendency in that direction. Further comparisons could be given, but we trust the point has been made; the English expression "average number of targets killed" is not definitive until details of combat have been fixed, and the manner of fixing those details affects the mathematical expression to be used, and in the process, the weapons that are ultimately selected.

Expression 1) is the one actually used in NAVMOR, this being deemed more appropriate than expression 2) because the environment is assumed to be target rich. Our vote is for the last expression. The fact that attrition is accounted for in the denominator does not excuse ignoring it in the numerator; attrition causes less targets to be killed as well as more aircraft to be lost. We choose 4) rather than 3) because the extra factor of Q can be thought of as a degradation factor for JMEM results, which do not deal explicitly with the effect of potential attrition on weapon delivery accuracy. However, 1), 3), and 4) are all essentially the same thing if Q is close to 1, as is typical of historical results.

Of more concern is the attrition scaling factor λ. It was mentioned in section 2.1 that PHASER provides an attrition goal to NAVMOR, and that NAVMOR adjusts λ until computed attrition meets the goal. To see the problem with the procedure in use, assume that the tactical choice is essentially one of being aggressive (in which case the target kill probability and the attrition probability are both large) or conservative (in which case both of those quantities are relatively small). The actual tactical choice might be altitude or dive angle. Assume further that the initial value for λ is such that aggregate attrition is too large. One would expect the system to react by using less aggressive tactics on the next pass, but in fact exactly the opposite will occur. Here is how it happens. Since attrition is too large, λ is adjusted downwards. The effect of this is to increase Q on the next pass, so the optimization procedure will select more aggressive tactics. In spite of the more aggressive tactics, less attrition will be measured, and the constraint will indeed be met if the procedure is repeated several times. The difficulty is that the adjustments of tactics are intuitively in the wrong direction. This is easiest to see in the extreme case where λ=0, in which

case the system would report no attrition at all while simultaneously using whatever tactics maximize damage to the target in one pass (pure aggression).

It might be argued that this "tactical instability" feature is not important because the NNOR is not used to determine tactics. However, the same issue exists for weapon selection, albeit not in such a pure form because weapons have a cost term in the denominator that does not vanish when $\lambda=0$. Suppose, for example, that NAVMOR were to produce a mix of standoff weapons (think of them as conservative weapons) and iron bombs (aggressive weapons) in response to given inputs from PHASER, and that the inputs from PHASER were changed to reflect a goal of having less attrition. The NAVMOR output could very well shift toward iron bombs! NAVMOR responds to requests for less attrition by convincing itself that attrition is less likely in the real world, with the natural result that the tactics and weapons selected become more aggressive.

The simplest way to fix the tactical instability feature would be to make attrition estimates that are absolute, rather than relative. The overall attrition could still be controlled to meet a constraint, if desired, by introducing a LaGrange multiplier on the cost of attrition. An increase in the multiplier would cause NAVMOR to produce less attrition by selecting more conservative tactics and weapons, rather than by changing its mind about the nature of the real world. This is easy enough to do mathematically; the difficult part of the fix is in getting absolute attrition estimates.

A final observation about NAVMOR is that it is biased toward special purpose weapons because it effectively assumes that weather, for example, is known chronologically before weapons must be loaded onto sorties. NAVMOR shares this property with the Air Force's FAST ATTACK program. Since section

3.1 contains a detailed discussion of the issue, there is no need to go into it here.

2.4 Observations on the THREAT Model

The THREAT model separately generates the combat requirements for each weapon under consideration by partitioning the threat into sets of targets applicable only to individual weapons. This forecloses the capability of the model to make weapons tradeoff or substitutability decisions on any basis. Such decisions must be made at a higher level of aggregation than that treated within the model and, hence, are not made on the basis of actual, microlevel combat substitutability considerations.

The threat partitioning, when combined with the "built-in" attrition aspect of the model, implies that targets serve no purpose other than to be destroyed. This is equivalent to the condition that attrition is independent of efforts to limit it. Even more extreme than NAVMOR, this means that different missions with the same weapon will result in probabilistically equivalent outcomes as long as only the target populations are the same. Related to this point, the model assumes that the only way that targets can avoid being destroyed, except when the maximum depth-of-fire has been reached, is by exhausting the ordnance of the platforms. This assumption ignores the effects of both leakers and saturation attacks on platform attrition.

The model assumes ordnance losses occur only through attrition of platforms. The logistics pipeline is assumed to be attrition-free, so that neither station ships nor depots suffer attrition losses. Along with the assumption of instantaneous resupply at refill events, this suggests that further work should be directed toward logistics considerations in the model if the model is to remain in use.

2.5 Ordnance Programming Model

The above models have provided no framework to compare alternative ordnance programs on the basis of costs and effectiveness. Because of this lack of an explicit methodology for programming of threat ordnance, the Center for Naval Analyses (CNA) was tasked to develop such a methodology. The resulting Ordnance Programming Model (OPM), see reference [7], provides a tool for trade-off analyses involving non-nuclear threat ordnance.

The general methodology of the OPM rests upon two points. The first is that the model must avoid the requirements, or threat-partitioning, approaches found in the NNOR in order that the substitutability of alternative weapons can be modeled. The second major point of the OPM is that each run begins with a specific inventory of ordnance, and the model then calculates the cost of that specific inventory as well as several measures of effectiveness. This approach enables explicit trade-offs to be made among alternative weapon inventories and programs.

Implementation of this methodology has resulted in a hierarchical model containing individual engagements which are aggregated up through missions, incidents, and theaters to define a scenario. The model is scenario-based and contains nine different predefined missions (AAW, ASW, and ASUW) plus user-defined missions as appropriate. There is an explicit handling of logistics and resupply considerations. The model is time-dependent and uses only expected values with accounting performed at the end of each period. The model has no wargaming capability and is run in a single pass through the scenario and its inputs. The following outcomes are available at the end of a scenario: number of targets killed, number of platforms surviving, quantity of ordnance fired, quantity of ordnance lost on attrited platforms, and

various user-defined measures of effectiveness which are functions of the previous outcomes (days of sustainability, survivability, etc.).

The specific methodology of OPM is implemented within engagements. Other levels of the model primarily provide accounting and input/output function. An engagement consists of a single type of target using one of four alternative return fire assumptions. These assumptions are (1) no return fire, (2) targets return fire only after platforms fire, (3) platforms and targets fire simultaneously with explicit modeling of return fire characteristics, and (4) simultaneous fire with a simple exchange ratio.

Engagements are assumed to be built up from Bernoulli trials and, hence, use the geometrical probability law in calculations. These calculations yield four outcome measures for each engagement: the number of targets killed, the number of platforms lost, the number of weapons expended, and the number of weapons lost on attrited platforms.

The number of targets killed is defined as

$$\text{TARGETS KILLED} = P_d*P_a*T*P_k,$$

where

P_d = Probability of detection = Pr(detection),

P_a = Pr(attack | detection),

T = number of targets, and

P_k = Pr(kill | attack).

The first three terms yield the number of targets attacked.

The probability of kill given attack, P_k, is calculated from

$$P_k = 1 - (q_k\ (1 - (p_k*p_r))^{SA-1}),$$

where

$q_k = 1 - p_k$,

$p_k = Pr(kill \mid single\ salvo)$,

$p_r = Pr(reattack)$, and

SA = salvos available per target attacked.

This probability is built on the single-salvo and not the single-shot kill probability. This formulation also assumes that the platform fires the number of salvos which are available on the platform.

The number of salvos available per target attacked, SA, is

$$SA = (PE*PL) / (P_d*P_a*T*(1-FA)*SS) ,$$

where

PE = platforms engaged,

PL = platform loadout,

FA = false attack ratio, and

SS = salvo size.

In any given engagement, SA may have to be adjusted due to changes in ordnance remaining on the platform. If SA is more than one but less than the maximum number that can be fired considering the kinematics of the engagement, then the value computed above is used without adjustment. If the computed value is greater than the maximum, it is reduced to the maximum. If SA is less than one, it is set to one and the number of targets attacked is reduced in the above equation to yield an SA of one. The only other alternative when SA is less than one is to reduce the salvo size, and this was rejected on operational grounds.

Total ordnance fired is calculated from

TOTAL ORDNANCE FIRED = $P_d*P_a*T*(1-FA)*SS*SF$,

where

SF = expected salvos fired per target attacked.

This formulation assumes that some targets which are fired upon are either false or outside of the correct firing envelope for the particular weapon. Expected salvos fired per target attacked, SF, is

$$SF = (1-(q_k*p_r)^{SA}) / (1-(q_k*p_r)).$$

Total ordnance lost is computed

TOTAL ORDNANCE LOST = (TOTAL PLATFORMS LOST)*(PL)*((1-RR)/2)

where

RR = platform reserve rate.

The number of platforms lost can be calculated using any of the four assumptions on return fire. This formulation assumes that platforms have an "average" magazine load which is midway between loadout and their reserves just before loadout. The amount of loadout depends upon the amount of ordnance of that type within the theater.

On a given platform, substitution can occur within weapons of the same general type since longer range weapons are assumed to be used up first and shorter range weapons substituted during a given mission. Note that platform attrition is modeled as an integral part of each engagement.

Resupply is addressed by the OPM through first deducting the ordnance in the maintenance pipeline. Rear echelon stocks are available across theaters, but there is no attrition of the rear echelon stocks. These stocks are attrited during inter-theater (once) and intra-theater (twice) moves. Attrition in resupply can occur to the ordnance but not to the resupply ships. Platforms can be replenished only between missions, with submarines handled as

a special case. OPM does not address the scheduling of ordnance since within theater ordnance is available instantaneously.

Criticisms of the OPM revolve around some of its resupply assumptions and the validity of some of its calculations in certain circumstances. Integration of more realistic assumptions into several components of the model could include the addition of intra-theater scheduling and attrition of resupply ships. Although the model is claimed to be an expected value model, it is not clear that some of the calculations detailed above can be carried out for nonintegral values of some of the parameters.

It should be noted, however, that the OPM is the only model we encountered which attempts to directly answer our first question posed above concerning the cost-effectiveness of alternative weapons mixes. Although imperfect, it offers the capability to make cost-effective weapon trade-off decisions.

3. Current Air Force Methods (Sabre Mix)

Like the Navy, the Air Force has distinct models for air-to-air and air-to-ground munitions. Only the air-to-ground methods will be reviewed here.

The Air Force's method for programming air-to-ground munitions is the Sabre Mix collection of models. The methodology of Sabre Mix is very similar to the Navy's NAVMOR. The keystone of Sabre Mix is HEAVY ATTACK (more recently known as FAST ATTACK), which is a close logical descendant of a model constructed by Clasen, et al [2]. The original model assigned aircraft sorties to targets "optimally" in a single period war, with the crucial inputs being

p_{ij} = average number of type j targets killed by a type i sortie, and with the crucial outputs being

s_{ij} = number of sorties of type i assigned to targets of type j.

The rest of the models in Sabre Mix are designed to make HEAVY ATTACK usable in a multi-period war where p_{ij} depends on weather and weapons carried and where the desired output includes information about munition consumption. Formally, if one is given the numbers

p_{ijkw} = average number of type j targets killed by a type i sortie loaded with weapon type k when the weather is w,

the problem is to somehow "process" all those numbers to obtain the p_{ij} inputs to HEAVY ATTACK, as well as to recover weapon usage from the output. The processing is performed by WEAPONEER (which produces the p_{ijkw}) and SELECTOR (which turns p_{ijkw} into p_{ij} and also provides attrition estimates). These three models constitute Sabre Mix. The passage of time is handled by simply running HEAVY ATTACK once in each time period. See Lord [3] for further details.

Subsequent sections will discuss specific aspects of this process that lead to bias or inaccuracy. Some of these problems are due to ultimate reliance on a model (HEAVY ATTACK) that seemingly does not have enough decision variables. Why not simply introduce the variables

s_{ijkwt} = number of sorties of type i loaded with weapon type k assigned to targets of type j in weather type w in time period t

and solve one big optimization problem, rather than trying to use the aggregated, myopic Sabre Mix process? The idea has been proposed and even tried. Hartman [1] gives a concise description of the TAC RESOURCER model, which does exactly that, and Lord [3] decribes how HEAVY ATTACK could be modified along those lines. The Air Force continues to rely on Sabre Mix, however. One major reason for this is no doubt solution time, since, of

course, models with more variables take longer to solve. Improved techniques and hardware will eventually mitigate this problem. Other reasons for this continued reliance are the expanded data base required for a more detailed model and a desire for year-to-year consistency.

The following sections deal with a specific parts of the Sabre Mix methodology. We will make no attempt to deal with grand questions such as "Can any approach based on a priori assessment of target values succeed?", "Why is it that the enemy has no choices here?", or "Why is least cost per kill the proper criterion for choosing weapon loads?" These questions have been raised in other places (Defense Science Board [12] and Bracken, et al [8]). Instead, we will carefully examine some of the inner workings of the Sabre Mix methodology.

3.1 Randomness and Optimization

Warfare is essentially unpredictable on account of the enemy's motivation to make it so. That can't be helped without employing Game Theory, which would greatly complicate the model and the optimization problem. Unintelligent sources of randomness can be and are taken into account in Sabre Mix, however. The two most important sources are:

a) the unpredictability of the results of a given sortie, even knowing the weather, and

b) the weather.

The first source of randomness is handled by ignoring it, which is reasonable because individual sources of randomness have a tendency to average themselves out when computing the objective function "total target value killed." Since p_{ij} is itself an average, the same thing is true of the objective function.

The second source of randomness is handled in SELECTOR by calculating

$$p_{ij} = \sum_{w} p_w \max_{k} p_{ijkw},$$

where p_w is the probability that the weather will be in one of the six possible "bands". This assumption tends to have the weapon type always optimized for the weather (the actual optimization is on cost per kill, rather than on p_{ijkw} directly, but that feature is not important for the moment). An alternative would be to compute

$$p_{ij}' = \max_{k} \sum_{w} p_w p_{ijkw},$$

which would correspond to the assumption that the true nature of the weather will be discovered <u>after</u> the choice of weapon type is made. The current (p_{ij}) method will load specialized weapons compared to the alternative, which will tend to select weapons that are robust to weather type. If this alternative assumption is true for a significant portion of the time in actual combat, the current method is biased to that extent toward specialized weapons.

Just as Sabre Mix assumes that weather is known <u>before</u> weapons are selected, it also assumes the weather is known only <u>after</u> aircraft are assigned to targets; this is manifest because the inputs p_{ij} to HEAVY ATTACK are not subscripted for weather. Sabre Mix must therefore be suspected of having a bias toward robust aircraft (actually robust aircraft-target assignments), since fair weather aircraft are in effect forced to fight in foul weather conditions with the proper climatological frequencies. This would be more worrisome if Sabre Mix were used to recommend aircraft inventories, since all-weather aircraft are over-valued by the procedure. It is not clear what effect this feature has on weapon selection, but there very well could be one.

The point is that sometimes aircraft can be assigned to targets considering the weather, and that sometimes weapons must be assigned to aircraft without knowing weather (or knowing it incorrectly). Sabre Mix ignores both of these possibilities, and is therefore biased in favor of robust aircraft and against robust weapons. There is no simple cure for this problem, although the magnitude of the weapon bias could at least be tested by using p_{ij}' (instead of P_{ij}) in HEAVY ATTACK and comparing results. Lord [3] also proposed running HEAVY ATTACK six times, once for each weather band. This would be a test of the aircraft bias.

3.2 HEAVY ATTACK objective function

The objective function in HEAVY ATTACK contains a "tuning parameter" C_j for each target type that has something to do with the extent to which dead targets can be distinguished from live ones. The parameter is not physically motivated, and the obvious way of motivating it would lead to the use of an objective function that differs from the one currently in use. The purpose of this section is to derive this new objective function and comment on the significance of the issue.

The problem can be illustrated with a one-period war where there are T identical targets that are gradually killed by sorties flown throughout the period. A fraction, 1-C, of the targets that are killed are assessed as such. This statement is the physical motivation mentioned earlier, since it is testable. The rest (C) serve to dilute the efforts of the aircraft in causing subsequent attrition, since they appear to remain alive and will continue to do so even if re-attacked.† If the number of dead targets is Y, the probability that any surviving target is chosen from the T-Y+CY that appear to be surviving

† A similar analysis can be carried out if the re-attack process may reveal apparently alive targets to be actually dead. The HEAVY ATTACK formula corresponds to neither assumption.

is the ratio (T-Y)/(T-Y+CY). Now let X be the number of targets that would be killed if there were no dilution; X is essentially a scaled number of sorties, and is given in HEAVY ATTACK by $\sum_i \sum_j P_{ij} S_{ij}$. The rate at which Y changes per unit increase in X is just the probability that a live target is attacked, which in turn depends on Y:

$$\frac{dY}{dX} = \frac{T-Y}{T-Y+CY} ,$$

an ordinary differential equation that can be solved by separating variables.

The solution is

$$X/T = (1-C)(Y/T) - C \ln(1-Y/T)$$

The formula relating Y to X in HEAVY ATTACK is $Y/T = (1- \exp(-CX/T))/C$, which is not the same thing. The two formulas agree when C = 1 (in which case both are $Y/T = (1-\exp(-X/T))$ or when C = 0 (in which case both are Y = X after L' Hopital's rule is applied to the HEAVY ATTACK formula). In general, however, there are differences, as can be seen in Figure 2, where the HEAVY ATTACK (HA) formula is compared with the new (NPS) formula:

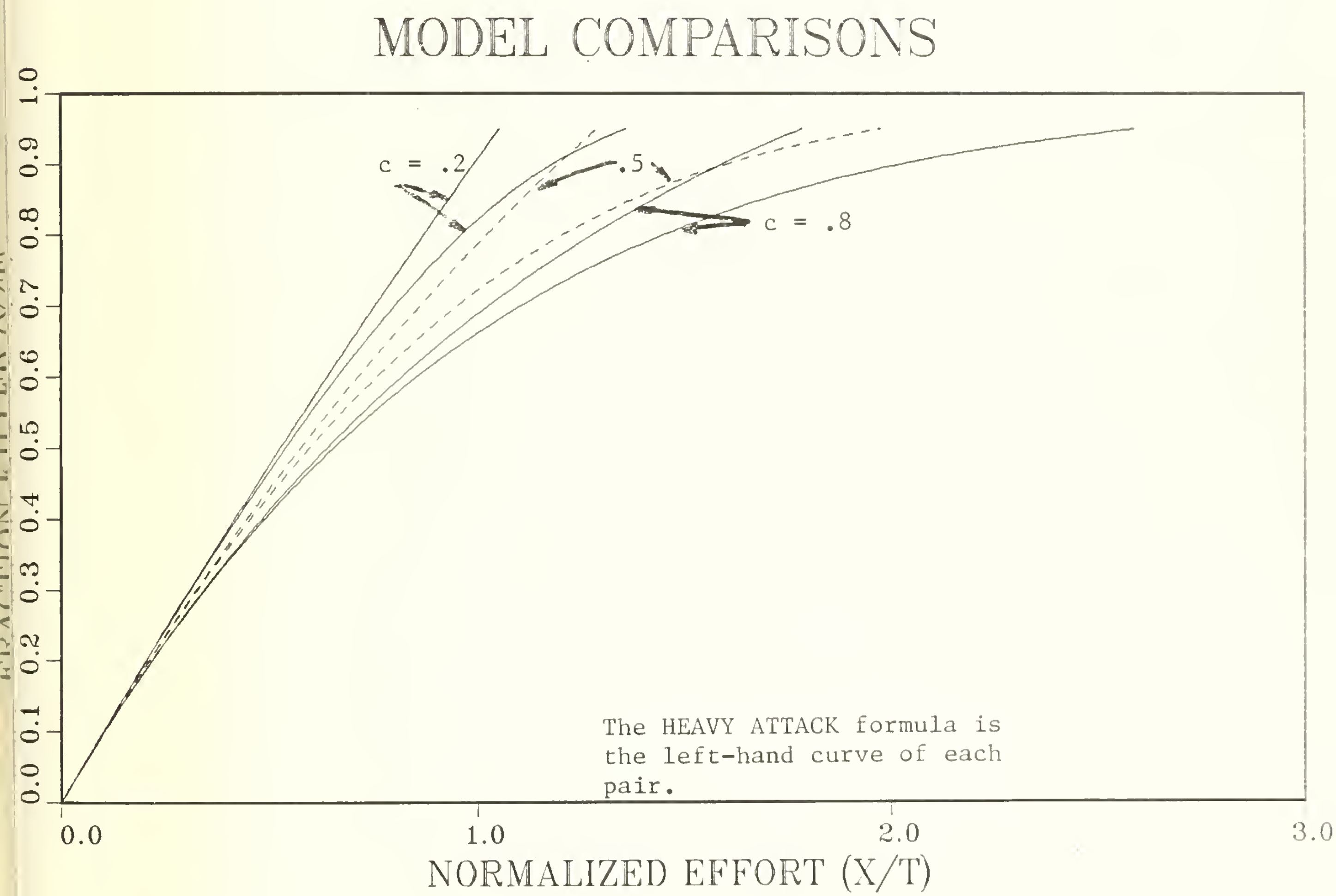

Figure 2

Figure 2 makes it clear that HA differs significantly from NPS, requiring less effort for a given fraction killed. The difference, while significant, may not be important. An argument for unimportance is that in practice C is generally set to one of the extremes 0 or 1, which are exactly the two cases where the formulas agree. A counter-argument is that nobody has ever before been sure what C represented except in those two cases, so that the prevalence of 0 and 1 in inputs does not indicate anything about actual battle conditions. Another argument for unimportance is that allocation of given assets is probably insensitive to the objective function anyway. The counterargument is that Sabre Mix can also be used to investigate the size of the required asset base, for which purpose the objective function is more important.

In summary, the Sabre Mix objective function currently involves a parameter C that has never been described physically. If the meaning of C is taken to be "fraction of killed targets that function as decoys", the NPS formula results, rather than the HA formula. Replacing HA with NPS would be significant in some cases.

4. Brief Review of Other Proposed Methods

4.1 Special Versus General-Purpose Weapons

Mangel and Nickel [11] express concern that current methods are biased in favor of special purpose weapons, the difficulty being that current methods do not give general purpose weapons sufficient credit for their greater flexibility in the face of uncertainty about target type, number, and order of arrival (see section 3.1). They propose a two-stage optimization procedure for properly handling these uncertainty issues. In the first stage, one calculates the best employment of a variety of weapon mixes against a variety of target sets, together with the resultant payoff. In the second stage, the weapon mix is selected to maximize the expected payoff, utilizing the assumed known probability distribution over target sets.

Symbolically, if $V(\{N_i\}|T)$ is the payoff when weapon mix $\{N_i\}$ is applied optimally to target set T, then it is $E_T(V(\{N_i\}|T))$ that should be optimized, where E_T represents expectation with respect to target set. Current procedures optimize $V(\{N_i\}|E_T(T))$, which is simpler computationally but essentially the wrong thing to do.

Optimizing $V(\{N_i\}|E_T(T))$ is an example of "expected-value-analysis". One simply proceeds as if a random quantity (T, in this case) could be replaced by its mean without affecting the conclusions of the analysis. The procedure is faulty, but has two things to recommend it:

a) One does not really "replace" a random variable by its mean. The variable involved is never thought of as being random at all. This at has the virtue of minimizing requirements for data collection, since no probability distribution is required.

b) Computational requirements are minimal, since only one value of the quantity need be considered.

The question is whether the gains in the quality of the answer are worth the costs of data collection and computation. For the weapon selection (stage 2) and allocation (stage 1) problem that they consider, Mangel and Nickel present evidence that expected-value-analysis is costly in terms of bias in the weapon mix ultimately selected. They also discuss some methods for minimizing the computational requirements of the "correct" method, amongst them being the use of heuristics in stage 1 and of stochastic approximation in stage 2. For problems on the scale of those considered by NAVMOR or Sabre Mix, however, the computational burden would still (we think) be prohibitive.

The issue of special versus general purpose weapons is also considered in Johnson and Loane [13].

4.2 RAF Model

Daniel, et al [14] report a conventional weapon procurement model that deals with the flexibility issue without introducing any probability distributions. Their method is Linear Programming (LP). The LP that they use is reproduced below:

objective function

$$\text{Maximize } \alpha r - \gamma \sum_k d_k + \delta \sum_{ik} (E_{ik} u_{ik}) \quad (\alpha,\ \gamma \text{ and } \delta \text{ weights},\ \gamma > \alpha > \delta)$$

Subject to:

Production capacity

$$n_i \leqslant A_i \quad \forall i$$

Budget limit

$$\sum_i C_i n_i \leqslant Q$$

Storage constraint

$$\sum_i S_i h_i \leqslant B$$

Flexibility constraint

$$r - d_k - \frac{1}{T_k} \sum_i E_{ik} \{u_{ik} + v_{ik}\} \leqslant -1 \quad \forall k$$

Basic sorties constraint

$$\sum_i u_{ik} + T_k d_k = T_k \quad \forall k$$

Flexible sorties constraint

$$\sum_i v_{ik_o} \leqslant \sum_{k \neq k_o} P_k T_k \quad \forall k$$

Weapon pool constraint

$$h_i - \sum_k u_{ik} - w_i \geqslant 0 \quad \forall i$$

Flexible weapon constraint

$$w_i - v_{ik_o} + \sum_{k \neq k_o} P_k u_{ik} \geqslant 0 \quad \forall i, k_o$$

Total weapon stock

$$h_i - n_i = L_i \quad \forall i$$

Variables

d_k sortie shortfall in role k (per planned sortie). If insufficient aircraft loads of weapons are available to meet the planned sortie allocation to roles, then $d_k T_k$ is the shortfall in sorties in role k. T_k is defined below. (Note that one aircraft load of weapons is needed for one sortie.)

h_i total number of aircraft loads of weapon type i held in stockpile.

n_i number of aircraft loads of weapon type i purchased.

r flexibility of the solution. If the maximum possible number of resources are concentrated on a given role, this gives a particular increase in effectiveness in that role above the planned level. The minimum value of this, over all roles, is the value of r.

u_{ik} number of aircraft loads of weapon type i used to satisfy the planned sortie allocation in role k.

v_{ik} number of aircraft loads of weapon type i (in excess of those to satisfy the planned sortie allocation) allocated to role k.

w_i number of aircraft loads of weapon type i held in a pool for flexibility.

Coefficients and constants

A_i production capacity of weapon type i (in aircraft loads).

B total storage volume available.

C_i cost of an aircraft load of weapon type i.

E_{ik} effectiveness of weapon type i in role k.

L_i initial stock level of weapon type i (in aircraft loads).

P_k proportion of planned sorties in role k which can be switched from this role.

Q total budget available.

S_i storage volume occupied by one aircraft load of weapon type i.

T_k planned number of sorties allocated to role k.

Note that the objective function is essentially heirarchical. The shortfalls d_k will be forced to 0 if possible because γ is so large, next r will be made as large as possible consistent with the d_k being 0, etc. The basic idea is that sufficient weapons must be bought to supply the planned sorties, with subsequent deviations from the plan being handled from a pool of flexible weapons. The flexible weapons have two sources: those purchased for the purpose (w_i) and those obtained by transferring weapons from other roles. Note that if the transfer fractions P_k were variables, rather than data, the optimal values would all be whatever maximum is permitted. Note also that aircraft do not enter explicitly, serving merely as the units in which weapons are measured.

The authors report that the model behaves sensibly in several respects. As the budget level Q is reduced, for example, the effect is to replace expensive, effective but specialized weapons with cheaper, more versatile but less effective weapons, while the total weapon stock remains constant over a broad range of Q. The model appears to have been well received. The authors state that "This approach appears to have been successful in answering immediate Air Staff problems and can certainly be claimed to have influenced decisions. It is, of course, too soon to assess whether the method, or a derivative, will be used more widely."

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APPENDIX ONE

STOCKPILE SEMINAR ATTENDEES

Al Washburn	NPS Faculty, OR	AV 878-2381
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Gil Howard	NPS Faculty, RA	AV 878-2098
Jim Hartman	NPS Faculty, OR	AV 878-3215
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Jerry Brown	NPS Faculty, OR	AV 878-2140
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